LILAVATIBAI PODAR HIGH SCHOOL (ISC)

PRELIMINARY EXAMINATION (2024-2025)

Grade: XII

Subject: Mathematics

Time: 3 hours | Marks: 80

Instructions to Candidates:

- You are allowed an additional 15 minutes for only reading the paper.
- You must NOT start writing during reading time.
- The question paper has 10 printed pages.
- The Question Paper is divided into three sections and has 22 questions in all.
- Section A is compulsory and has fourteen questions.
- You are required to attempt all questions either from Section B or Section C.
- Section B and Section C have four questions each.
- Internal choices have been provided in two questions of 2 marks, two questions of 4 marks and two questions of 6 marks in Section A.
- Internal choices have been provided in one question of 2 marks and one question of 4 marks each in Section B and Section C.
- While attempting Multiple Choice Questions in Sections A, B and C, you are required to write only ONE option as the answer.
- The intended marks for questions or parts of questions are given in the brackets [].
- All workings, including rough work, should be done on the same page as, and adjacent to, the rest of the answer.
- Mathematical tables and graph papers are provided. Instruction to Supervising Examiner
- Kindly read aloud the instructions given above to all the candidates present in the examination hall.

Section A

Question 1: [15]

i. Which of the following statement explain a determinant correctly

a. Determinant is a number associated to a matrix
b. Determinant is a square matrix
c. Determinant is a number associated to a square matrix
d. Determinant is an ordered array of elements

ii. If $\int x^6 \sin(5x^7)\,dx = \frac{k}{5}\cos(5x^7) + C$, then the value of k is

a. $-1/7$
b. $1/7$
c. $1/35$
d. $-1/35$

iii. If a function $f: Q \to R$ is defined by $f(x) = \frac{2x-1}{2}$ and function $g: Q \to R$ is defined by $g(x) = x + 2$, then $(gof)\left(\frac{3}{2}\right)$ is

a. 1
b. 2
c. $\frac{7}{2}$
d. none of these

iv. Assertion: The degree of differential equation $a\left(\frac{dy}{dx}\right)^2 + b\left(\frac{dx}{dy}\right) = c$, is 3

Reason: If each term involving derivatives of a differential equation is a polynomial (or can be expressed as a polynomial) then the highest exponent of the highest order derivative is called the degree of the differential equation.

a. Both A and R are correct and R is the correct explaination of A
b. Both A and R are correct but R is not the correct explaination for A
c. A is correct but R is incorrect
d. R is correct but A is incorrect

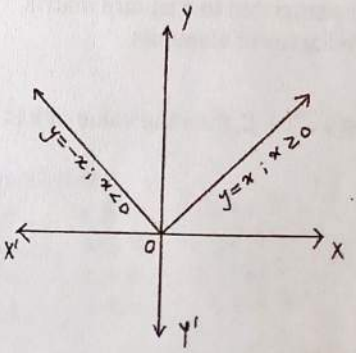
v. If $P(A) = \frac{3}{10}$, $P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$, then $P(B|A) + P(A|B) =$

a. $\frac{1}{4}$
b. $\frac{1}{3}$
c. $\frac{5}{12}$
d. $\frac{7}{12}$

vi. If A is a square matrix of order 3 and $|A| = 2$, then the value of $|-AA^T|$ is

a. 4
b. 2
c. -2
d. -4

vii. The graph of the function $f: R \to R$ defined by $f(x) = |x|$ is given below:



The range of the function is

a. $[0, \infty)$
b. $(0, \infty)$
c. $(-\infty, \infty)$
d. $R - \{0\}$

viii. For the same graph given in the above question vii, the given function $f(x)=|x|$ is
a. injective
b. surjective
c. bijective
d. many – one into

ix. Assume that in a family, each child is equally likely to be a boy or a girl. A family with three children is chosen at random. The probability that the eldest child is a girl given that the family has at least one girl is
a. $\frac{1}{2}$
b. $\frac{1}{3}$
c. $\frac{2}{3}$
d. $\frac{4}{7}$

x. Assertion (A): The determinant of a skew-symmetric matrix of odd order is zero
Reason (R): For a skew symmetric matrix $A : A' = -A$
a. Both A and R are correct and R is the correct explaination of A
b. Both A and R are correct but R is not the correct explaination for A
c. A is correct but R is incorrect
d. R is correct but A is incorrect

xi. A speaks the truth 3 out of 5 times while B speaks the truth 4 out of 7 times. If a fair coin is tossed once and shown to A and B discretely, in what percent of the cases will they vary from each other.

xii. If $A = \begin{bmatrix} 2 & 1 \\ 7 & 5 \end{bmatrix}$, without finding A(adjA), find the value of |A. adjA|

xiii. If a function f is defined by $f: R \to R$ is defined by

$$f(x) = \begin{cases} 2x, & x > 3 \\ x^2, & 1 \le x \le 3 \\ 3x, & x \le 1 \end{cases}$$

Find the value of $f(-1) + f(2) + f(4)$

xiv. If the domain of $\cos^{-1} x$ is $[-1,1]$ then find the domain of $\cos^{-1}\sqrt{1-x}$

xv. Find the gradient i. e. $\frac{dy}{dx}$ at (1,4) for $\sqrt{x} + \sqrt{y} = \sqrt{c}$

Question 2: [2]

a Differentiate the following function w. r. t x:

$$\tan^{-1}\sqrt{\frac{1+\cos x}{1-\cos x}}$$

Tried left midway

OR

b If an edge of a variable cube is increasing at the rate of 0.5 cm/sec, at what rate is the surface area increasing when the edge is 12cm?

Question 3: [2]

Evaluate: $\int_{-1}^{1}(x-[x])dx$ Tried

Question 4: [2]

It is known that the given function is continuous at $x = 2$.
Check whether it is differentiable at $x = 2$.

$f(x) = 2x^2 - x;\ 1 \le x \le 2$
$= 5x - 4;\ x > 2$

Question 5: [2]

a Evaluate: $\int \frac{e^{2\log x}+1}{e^{2\log x}-1}\,dx$

OR

b Arrange the given differential equation in the form $\frac{dy}{dx} + Py = Q$ and hence find the integrating factor for the following linear differential equation:

$x\frac{dy}{dx} + y - x + xy\cot x = 0$ Tried but not sure, got thru $\frac{1}{2}$ of it

Question 6: [2]

If $\tan^{-1}x - \cot^{-1}x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$, $x > 0$, then find the value of x.

Question 7: [4]

Solve for the value of x: $\sin^{-1}\left(\frac{x}{2}\right) + \cos^{-1}x = \frac{\pi}{6}$ Calc

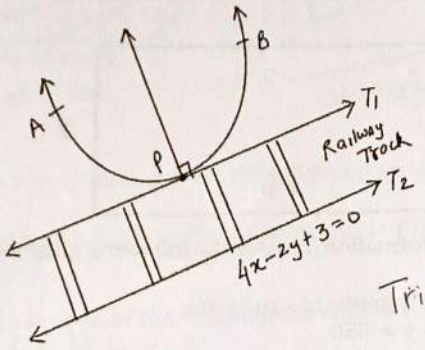
Question 8: [4]

Evaluate: $\int \frac{3x-2}{(x+1)^2(x+3)}\,dx$

Tried left

Question 9: [4]

a As shown in the given figure below there is a curve AB whose equation is given by $y = \sqrt{5x-3} - 2$. A cyclist is moving along the curve from point A towards point B. P is the point on the curve which touches railway track T_1 which is parallel to T_2.



Tried left

Based on the above information:

i. Find the slope of T_1

ii. Abscissa of point P on track T_1 x

iii. Ordinate of point P on track T_2 y

iv. Equation of track T_1

OR

b If $x = a\cos\theta + b\sin\theta, \quad y = a\sin\theta - b\cos\theta,$

show that $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0.$

Question 10: [4]

a Two friends Aman and Bipin are playing a game in which they throw a pair of coins alternatively and decided who gets both heads first will win the game. If Bipin starts the game, based on the above information, answer the following questions:

i. What is the probability that Bipin wins in his third attempt?

ii. What is the probability that Aman wins the game.

OR

b A building contractor has undertaken a contract job. The probability that there will a construction workers strike is 0.65. If there is a strike and still the construction work will be completed on time, the probability is 0.32. The construction work will be completed on time in the absence of any strike has the probability of 0.80

Based on the above information, find the probability that:

i. there will be no strike?

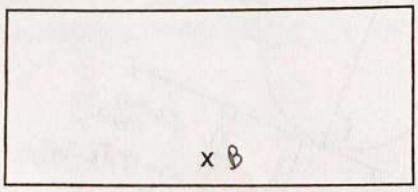
ii. construction job will be completed on time?

iii. if the construction job is completed on time, what is the probability of no strike?

Support your solution with a **tree diagram.**

Question 11: [6]

Ruhel wants to donate a rectangular plat of land for a school in his village. When he was asked to give dimensions of the plot, he said that if the length is decreased by 50m and breadth is increased by 50m, then the area will remain the same. But if the length is decreased by 10m and breadth is decreased by 20m, then the area will be decreased by 5300 sq. meters.



Based on the given information, answer the following questions:

i. Which is the equation in terms of x and y are:

a. $x - y = 50$, $2x - y = 550$
b. $x - y = 50$, $2x + y = 550$
c. $x + y = 50$, $2x + y = 550$
d. $x + y = 50$, $2x - y = 550$

got diff ans then mcq

ii. Represent the correct equation in $AX = B$ form

iii. Find the value of x (length of the rectangular field) and y $\left(\begin{matrix}\text{breadth of} \\ \text{rectangular field}\end{matrix}\right)$

iv. Hence, find the total area of land which Ruhel wishes to donate.

Question 12: [6]

a Evaluate: $\int_0^{\pi/2} \frac{\sin^2 x}{1 + \sin x + \cos x} dx$ Tried but got wrong I think

OR

b Show that the given equation is homogenous. Hence, find the particular solution of the differential equation:

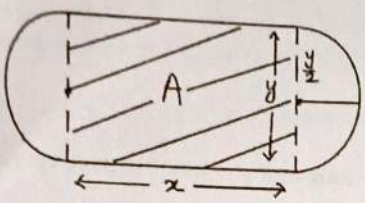
$\left(x \sin^2\left(\frac{y}{x}\right) - y\right) dx + x dy = 0$, given $y(1) = \frac{\pi}{4}$

Question 13: [6]

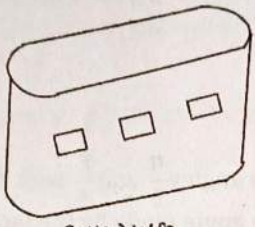
a If the sum of the surface areas of a cube and a sphere are constant, what is the ratio of an edge of the cube to the diameter of the sphere when the sum of the volumes is minimum?

OR

b An architect designs a building for a multi-national company. The floor consist of a rectangular region with semicircular ends having perimeter of 200m as shown in the below diagram depicting the Design of Floor.



DESIGN of FLOOR


BUILDING

Based on the information provided, answer the following questions:

i. If x and y represent the length and breadth of the rectangular region, then find the relation betwen x and y

ii. Form the equation for the **area of the rectangular region A** expressed as a function of x.

iii. Find the maximum value of Area A

iv. The multinational company is not interested to maximise the area of the **whole floor** i. e. the rectangular part along with the semi – circular part. For this to happen, what should be the value of x?

Question 14: [6]

In a game, Nikhil wins Rs. 8 for getting a number greater than 3 and loses Rs. 3 otherwise, when a fair die is thrown. Nikhil decided to throw the die 4 times but to quit as and when he gets a number greater than 3.

i. If X denotes the amount which Nikhil wins or loses, then find all the possible values of X.

ii. Find the corresponding values of X and make the probability distribution table.

iii. Find the expected value of the amount which Nikhil will win.

Section B

Question 15: [5]

In subparts (i) to (iii) choose the correct options and in subparts (iv) to (v), answer the questions as instructed.

i. The lengths of perpendicular drawn from the point $(4, -7, 3)$ on the $y-$ axis is

a. 3 units
b. 4units
c. 5 units
d. 7 units

ii. If $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 144$ and $|\vec{a}| = 4$, then $|\vec{b}|$ is equal to
a. 12
b. 3
c. 4
d. -3

iii. If a line makes angles $\frac{\pi}{3}$ and $\frac{\pi}{4}$ with the $x-$axis and $y-$axis respectively, then the acute angle made by the line with $z-$axis is
a. $\pi/6$
b. $\pi/4$
c. $\pi/3$
d. $5\pi/12$

iv. Find the area of the region bounded by the curve $x^2 = 4y$, the line $x = 2$ and $x-$axis.

v. Find the vector and the cartesian equations of the line through the points $(5, 2, -4)$ and which is parallel to the vector $3\hat{i} + 2\hat{j} - 8\hat{k}$.

Question 16:

a Find the value of p for which the following planes are perpendicular to each other: $\vec{r} \cdot (p\hat{i} + 2\hat{j} + 3\hat{k}) = 5$ and $\vec{r} \cdot (\hat{i} + 2\hat{j} - 7\hat{k}) + 11 = 0$ [2]

OR

b The equation of the line are $5x - 3 = 15y + 7 = 3 - 10z$. Write the direction cosines of the line.

Question 17:

Find the smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$. [4]

Question 18:

a Find the image of the line $\frac{x-1}{3} = \frac{y-3}{1} = \frac{z-4}{-5}$ in the plane $2x - y + z + 3 = 0$. [4]

OR

b Find the vector and the cartesian equations of a line through the point $(1, -1, 1)$ and perpendicular to the lines joining the points $(4, 3, 2)$, $(1, -1, 0)$ and $(1, 2, -1)$, $(2, 1, 1)$.

Section C

Question 19: In sub parts (i) to (iii) choose the correct option and in sub parts (iv) and (v), answer the questions as instructed [5]

i. The demand function of a monopolist is given by x=100-4p. The quantity at which MR=0 will be
   a. 25
   b. 10
   c. 50
   d. 30

ii. If the lines of regression are parallel to the coefficient of correlation is:
   a. 1
   b. 0
   c. −1
   d. 1/2

iii. If the total cost function is given by $C(x) = 10x - 7x^2 + 3x^3$, then the marginal average cost is given by
   a. $10 - 14x + 9x^2$
   b. $10 - 7x + 3x^2$
   c. $-7 + 6x$
   d. $-14 + 18x$

iv. The cost function C(x)for x breads is given by $C(x) = 3.5x + 12000$. If each bread is sold for Rs. 6, determine the minimum number of breads that should be produced and sold to ensure no loss.

v. The marginal revenue function of a commodity is given by $MR = 11 - 3x + 4x^2$. Find the revenue function.

[2]

Question 20:

a The total cost function of a production is given by $C(x) = \frac{3}{4}x^2 - 7x + 27$. Find the number of units produced for which MC = AC

OR

b The cost function associated with producing and marketing x units of an item is given by $C(x) = 2x^2 - 11x + 50$. Find the range of x for which AC is increasing.

[4]

Question 21:

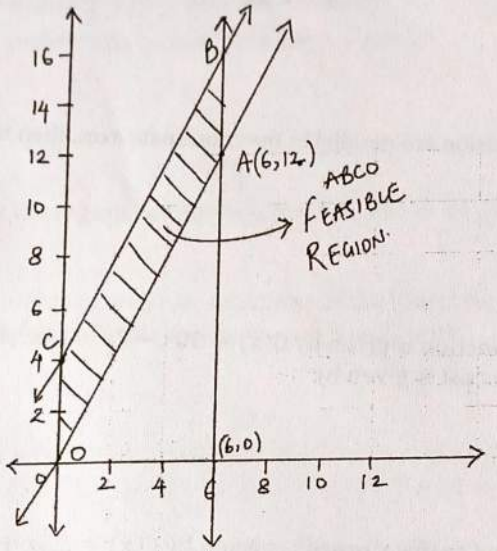
a In a bivariate distribution, the two lines of regression are 2x+3y-8=0 and x+2y-5=0 and variance of x is 12. Find:
   i) mean of x and y
   ii) identify the line of y on x
   iii) correlation coefficient
   iv) variance of y

OR

b For the given pairs of observation, find the required regresion line and estimate the value of y when x = 25.
(9, −4), (10, −3), (11, −1), (12, 0), (13, 1), (14, 3), (15, 5), (16, 8).

[4]

Question 22:
The feasible region for an LPP is shown in the adjoining figure.



Line CB is parallel to OA. Based on the given information, answer the following questions:

i) The equation of line OA is
a) $x - 2y = 0$
b) $y - 2x = 0$
c) $x + 2y = 0$
d) $2x + y = 0$

ii) The equation of line BC is
a) $y - 2x = 4$
b) $x - 2y = 4$
c) $y + 2x = 4$
d) $x + 2y = 4$

iii) The constraints for the LPP are
a) $y \geq 2x$, $y - 2x \leq 4$, $x \leq 6$, $x, y \geq 0$
b) $y \leq 2x$, $y - 2x \leq 4$, $x \leq 6$, $x, y \geq 0$
c) $y \geq 2x$, $y - 2x \geq 4$, $x \leq 6$, $x, y \geq 0$
d) $y \leq 2x$, $y - 2x \geq 4$, $x \leq 6$, $x, y \geq 0$

iv) Find the minimul value of the objective function $Z = 3x - 4y$